Government Polytechnic

Vaishali.

Mechanics of Structures

Sub code - 1615402

Semester-4

Unit -7 Strain Energy

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STRAIN ENERGY AND RELATED DEFINITION

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STRAIN ENERGY

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In physics, strain energy is the energy stored by a system undergoing deformation. For linearly elastic materials, strain energy is:

$$U = \frac{1}{2} V \sigma \epsilon = \frac{1}{2} V E \epsilon^2 = \frac{1}{2} \frac{V}{E} \sigma^2$$

where σ is stress, ε is strain, V is volume, and E is Young's modulus:

$$E = \frac{\sigma}{\epsilon}$$

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In material science, resilience is the ability of a material to absorb energy when it is deformed elastically, and release that energy upon unloading. Proof resilience is defined as the maximum energy that can be absorbed up to the elastic limit, without creating a permanent distortion.

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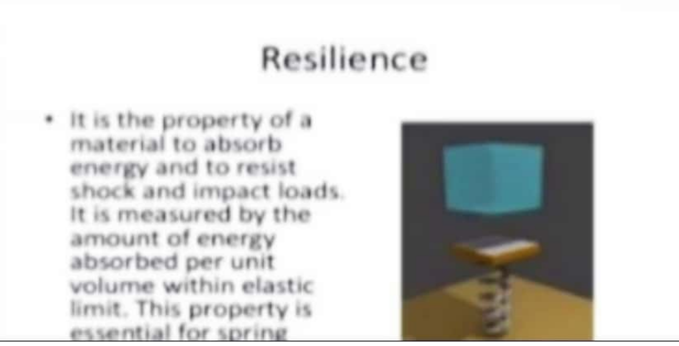
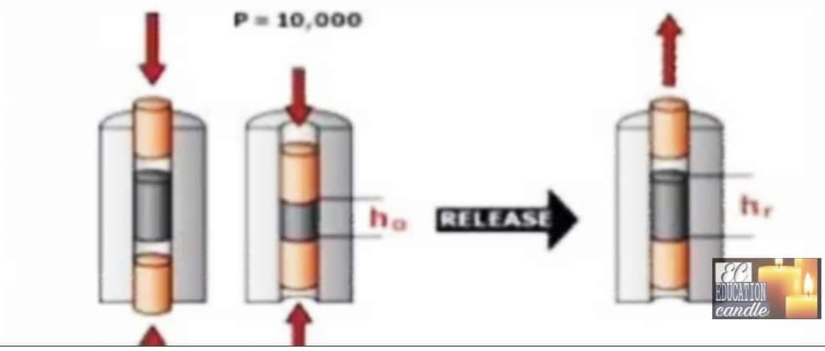
RESILIENCE:

- It is the capacity of a material to absorb energy elastically.
- The maximum energy which can be stored in a body upto elastic limit is called the ***proof resilience, and*** the proof resilience per unit volume is called ***modulus of resilience***.
- The quantity gives capacity of the material to bear shocks and vibrations.





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Proof load is defined as the maximum tensile force that can be applied to a bolt that will not result in plastic deformation. In other words, the material must remain in its elastic region when **loaded** up to its **proof load**. **Proof load** is typically between 85-95% of the yield strength.

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GRADUALLY APPLIED LOAD , SUDDENLY APPLIED LOAD AND IMPACT LOAD

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STRAIN ENERGY DERIVATION

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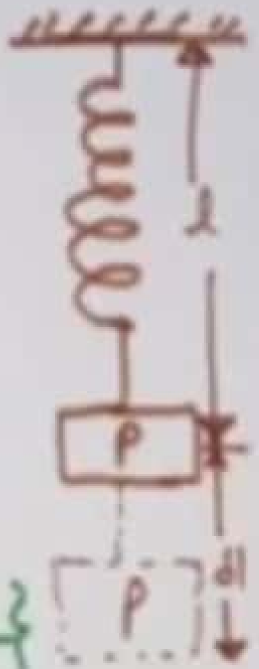
Education Candle

→ Strain Energy



Strain Energy = work done

- work done = Avr. Resilience × δl

$$= \frac{0+P}{2} \times \delta l$$

$$= \frac{P}{2} \times e \times l \quad \left\{e = \frac{\delta l}{l}\right\}$$

$$= \frac{\sigma \times A}{2} \times e \times l \quad \left\{\sigma = \frac{P}{A}\right\}$$

$$= \frac{\sigma \times e}{2} \times A \times l$$

$$= \frac{\sigma}{2} \times e \times V \quad \{V = A \times l\}$$

$$= \frac{\sigma}{2} \times V \times \frac{\sigma}{E} \quad \left\{E = \frac{\sigma}{e}\right\}$$

$$\boxed{\text{Strain Energy} = \frac{\sigma^2}{2E} \times V}$$

→ Proof Resilience $= \dfrac{\sigma_{max}^2}{2E} \times V$

→ Modulus of Resilience $= \dfrac{\sigma_{max}^2}{2E} \times \dfrac{V}{V} = \dfrac{\sigma^2}{2E}$

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GRADUALLY APPLIED LOAD

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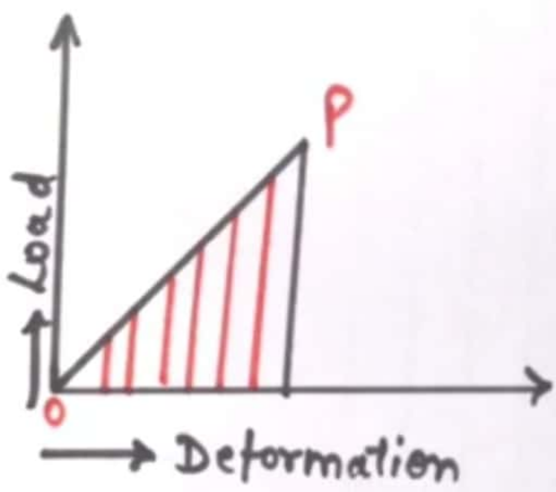
"Stress due to Gradually Applied Load"



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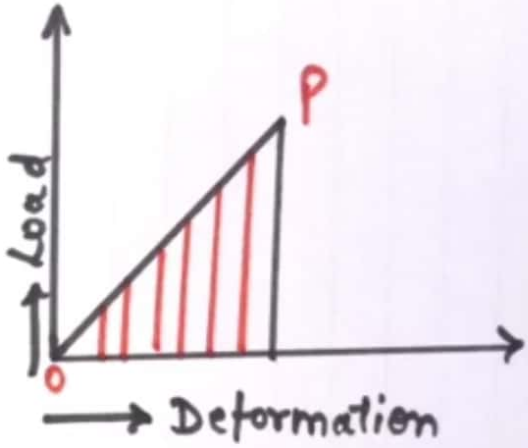
"Stress due to Gradually Applied Load"

→ Strain Energy = Area Under the Curve



As we know, $U = \frac{\sigma^2}{2E} \times V$

So that, $U = \frac{1}{2} \cdot P \times \delta L$

$$\frac{\sigma^2}{2E} \cdot V = \frac{1}{2} \cdot P \cdot \frac{PL}{AE} \quad \left\{ \delta l = \frac{PL}{AE} \right\}$$

$$\frac{\sigma^2}{2E} \times A \times L = \frac{1}{2} \cdot \frac{\sigma L}{AE} \cdot P \quad \left\{ \frac{P}{A} = \sigma \right\}$$

$$\boxed{\sigma = \frac{P}{A}}$$

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SUDDENLY APPLIED LOAD

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Sudden Load: It is the load without pre-effect and it may stay for the longer time period after the application of load. In other words it is the kind of unplanned or undesired load in most of the cases. This load imparts at random times and may disturb the system at a great intent. As it is unplanned load, the system might not be designed to bear that. It also do not have fixed value or direction, its on the roll of dice.

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STRESS DUE TO SUDDEN APPLIED LOAD

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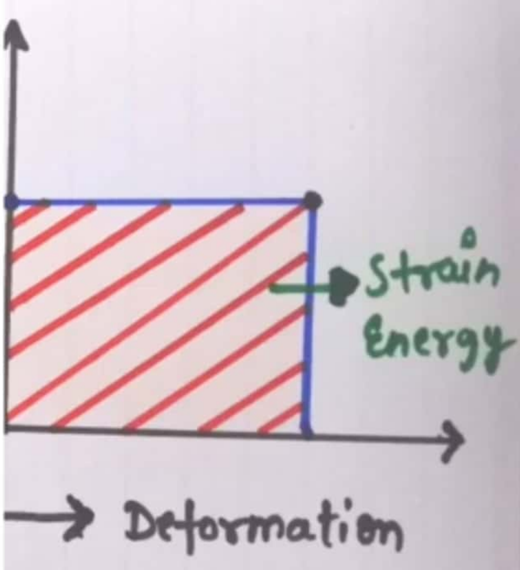
"Stress due to Suddenly APPLIED LOAD"



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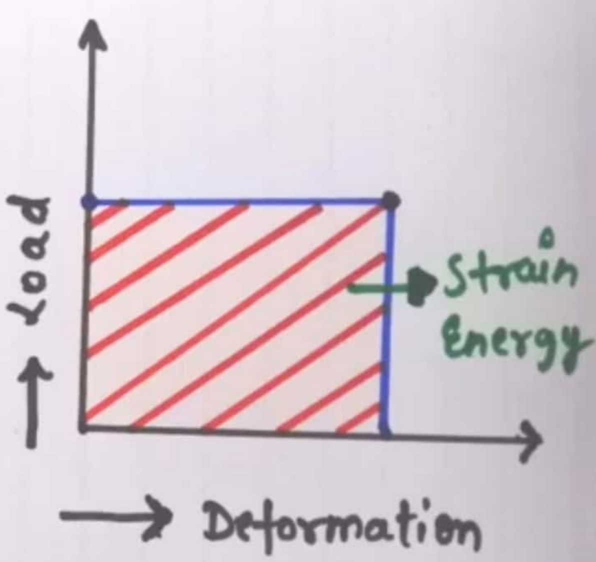
"Stress due to Suddenly APPLIED LOAD"



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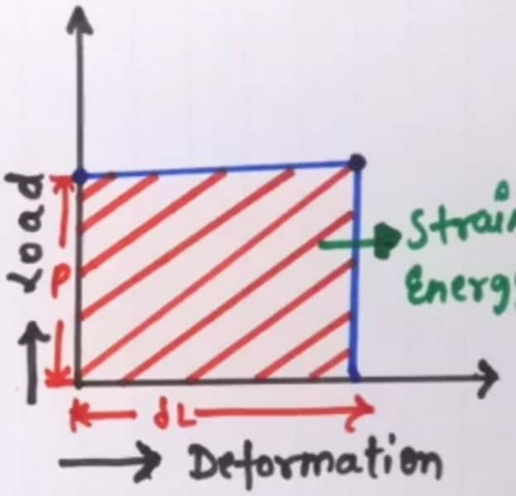
∴ Strain Energy = Area under the Curve

$$U = P \cdot \delta L$$

$$\because U = \frac{\sigma^2}{2E} \cdot V$$

$$\frac{\sigma^2}{2E} \cdot A \times L = P \times \frac{\sigma L}{E} \quad \left\{ \frac{P}{A} = \sigma \right\}$$

$$\boxed{\sigma = \frac{2P}{A}}$$



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STRAIN ENERGY DUE TO IMPACT LOAD

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Impact Load: In almost all the cases impact load is designed to be imparted on the system in a fraction of second. Impact load is designed so it has certain value and pre-defined line of action. It is been removed with immediate effect, unlikely to sudden load in some cases. In other words we can say that impact load is the designed version of sudden load, where we have designed the impact to test the system or to bear natural conditions.

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STRESS DUE TO IMPACT LOAD

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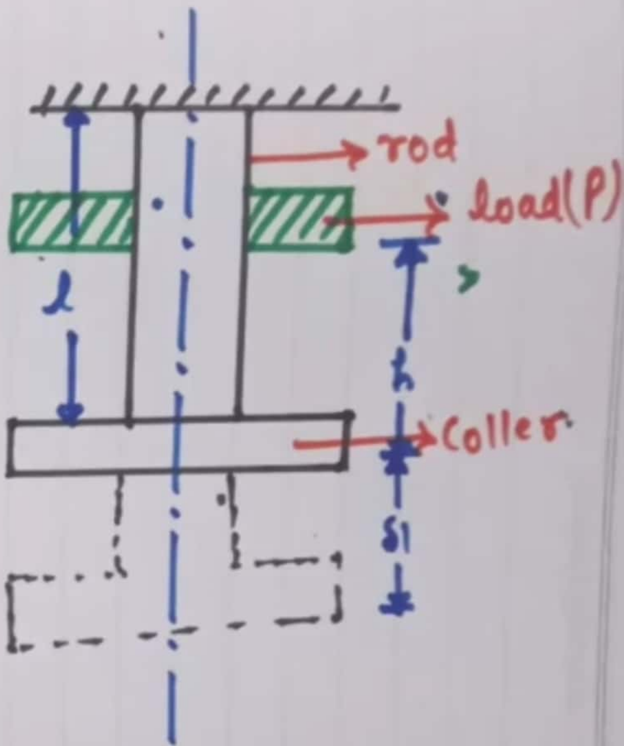
→ Deformation $(\delta l) = \frac{PL}{AE}$ — ①

ork done $= P \times (h + \delta L)$

$= P \times \left(h + \frac{PL}{AE}\right)$ — ②

∵ work done = Strain Energy

$$P \times \left(h + \frac{PL}{AE}\right) = \frac{\sigma^2}{2E} \times A \times L$$



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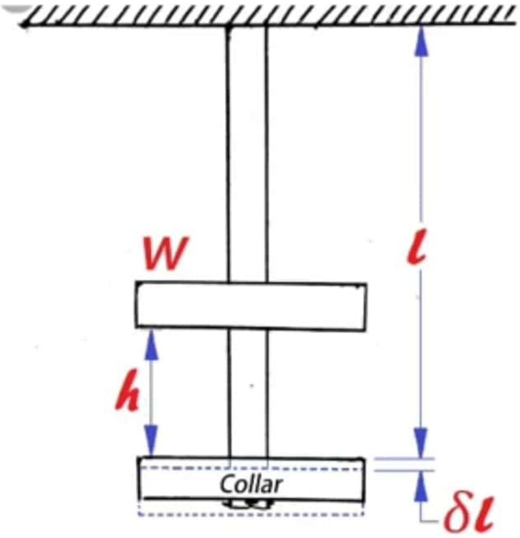
→ Deformation = δl

→ work done = $P(h+\delta l)$

Strain Energy = workdone

$$U = P(h+\delta l)$$



$$\frac{\sigma^2}{2E} \times A \times L = Ph + P\delta l$$

$$\frac{\sigma^2}{2E} \times A \times L - Ph - P\frac{\sigma L}{E} = 0$$

$$\sigma^2\left(\frac{AL}{2E}\right) - \sigma\left(\frac{PL}{E}\right) - Ph = 0$$

$$\sigma^2\left(\frac{1}{2}\right) - \sigma\left(\frac{P}{A}\right) - \frac{2PhE}{AL} = 0, \left\{\text{Multiply } E/AL\right\}$$

$$\sigma = \frac{P}{A} \pm \sqrt{\frac{P^2}{A^2} + \frac{2PhE}{AL}}$$

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